PREDICTION OF MERGE RATIO USING LANE FLOW DISTRIBUTION

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ABSTRACT

Real data from several freeway merges reveal that merge ratios can be effectively estimated by incorporating lane flow distributions (LFD). Based on the findings, two-stage models are developed, in which LFD are modeled statistically in terms of traffic conditions and geometric characteristics, and then the predicted LFD are used to estimate merge ratios. Validation results indicate that the two stage models based on LFD provide reasonable estimates of merge ratios. Nonetheless, model enhancement is desired to capture other influencing factors of LFD. Our results also indicate that the fair-share merge principle gives more accurate and consistent estimates of merge ratios than the zipper principle.

INTRODUCTION

Merges constitute important physical characteristics of freeways and have a significant impact on the performance of freeway networks, especially during congestion. At merges, drivers from conflicting traffic streams must compete to merge in a systematic manner, and their behavior is linked to some of the most important traffic phenomena such as capacity drop *(1-2)*. A number of studies have attempted to describe this behavior *(3-11).* Particularly, Daganzo *(5)* proposed a simple merge model where the merging behavior of two congested traffic streams is dictated by drivers taking turns to merge at a fixed ratio. This model is also consistent with Papageorgiou *(3)* in the special case of a triangular fundamental diagram. This "merge ratio" is more generally defined as the ratio of the inflows from two conflicting traffic streams when both approaches are congested. This parameter is a key component to predicting congestion propagation and evaluating freeway performance in sections upstream of merges. Several studies verified empirically that the relationship between inflows is indeed reproducible and linear, suggesting a constant merge ratio for a given site *(12-13)*. However, merge ratios were found to be site-specific, requiring direct field measurements or a predictive model.

Ni et al. *(10)* proposed a merging scheme where the merge ratio is described as the proportion of the upstream branches' capacities. They refer to this principle as *fair share* and argue that the capacity serves as a good representation of the downstream supply since it takes into account the number of lanes and per lane capacity. Since the capacity of a merging stream is mostly dictated by its number of lanes, the capacity ratio is intuitively similar to the lane ratio. Another premise suggested by everyday driving experience says that traffic from lanes adjacent to the merge compete to merge on a one-to-one basis (referred to as the "zipper rule") while the flow of the other lanes remains constant.

Bar-Gera and Ahn *(13)* conducted a macroscopic study of merge ratios using data from fifteen different sites. They found that lane ratios provide a reasonable estimate of merge ratios in the absence of field data. Nevertheless, significant residuals suggested that other factors influence merge ratio. It is important to note that a lane ratio does not account for variations in lane-wide capacities or utilizations at different flow levels. On the contrary, studies have found that lane flow distributions (LFD) vary significantly across lanes with respect to the total freeway flow in both congested and uncongested states *(15-22)*. Thus, it is likely that LFD near a merge significantly affect the merge ratio.

Numerous studies have found reproducible patterns of LFD with respect to flow and density. In the typical LFD relationship, the proportion of flow in the median lane increases as the total flow increases, while the proportions of flow in the other lanes, particularly the shoulder lane, decrease for both congested *(15-18)* and free flow conditions *(19-20).* Similar patterns of LFD were observed with respect to density *(17-19).* However, the observed trends were rather mixed, depending on the traffic conditions studied (e.g. high flow, low flow or uncongested and congested); some report linear trends *(17, 20-22)* while others report non-linear trends *(17-19, 21)*.

LFD is shown to vary significantly between sites *(15-16),* even in sites that exhibit similar geometry *(17)*. Also, time of the day was found to have a significant impact on LFD patterns *(15),* whereas day of the week was found to have no impact *(16)*. These variations are not surprising and can be attributed to transitions from uncongested to congested states throughout a day and site-specific recurrent congestion patterns. Additionally, it was found that traffic controls can significantly impact LFD. For instance, variable speed limits (VSL) significantly affect LFD, especially at high flow and density values, by improving the utilization of the shoulder lane *(19-20)*.

Hong et al. *(21)* separated passenger cars and heavy trucks and modeled LFD using non-linear regression for two and three lane freeways for each vehicle type. They found that vehicle type, lane type (median, center or shoulder), and rainfall have a significant impact on LFD. Wu *(18)* developed a probabilistic model to predict LFD for two, three, four, and five lane facilities and used data from a freeway in Germany to validate the model. To reduce the complexity of implementing the probabilistic model, he generated exponential regression models that mimic the probabilistic trend of the data with five parameters; the data fitted well the model. Additionally, Lee and Park *(17)* used polynomial regression to the third degree with density ratio as the predictor for 2 and 3 lane models, and Hurdle et al. *(22)* described a negative linear relationship of LFD with respect to flow with non-linear relationships at the tails. These research efforts have served to provide valuable insight of lane specific traffic patterns and to understand the traffic characteristics that influence LFD. Yet, the models present several drawbacks: (i) they are very complex and thus hard to interpret and difficult to implement in practice, (ii) they were calibrated and validated with few sites, which poses a risk of modeling only local characteristics which may not be applicable to other locations. Therefore, there is a need for simpler general models of LFD with few observable variables, that are easy to interpret and that can be transferable to diverse locations with different traffic and geometric characteristics.

The objectives of this paper are: (i) to verify the relationship between LFD and merge ratio, (ii) develop models of LFD that are general, accurate and simple, (iii) to develop models that accurately reproduce the merging scheme of various sites using the intrinsic relationship of LFD and merge ratio, and (iv) to test two theoretical principles of merge ratio, the “fair-share” theory and the “zipper” rule. To this end, we develop two-stage models in which LFD is modeled in terms of traffic conditions and geometric characteristics (stage 1), and the predicted LFD is used to estimate the merge ratio (stage 2). We examine both merge principles and compare the performance against merge ratios based on lane ratios.

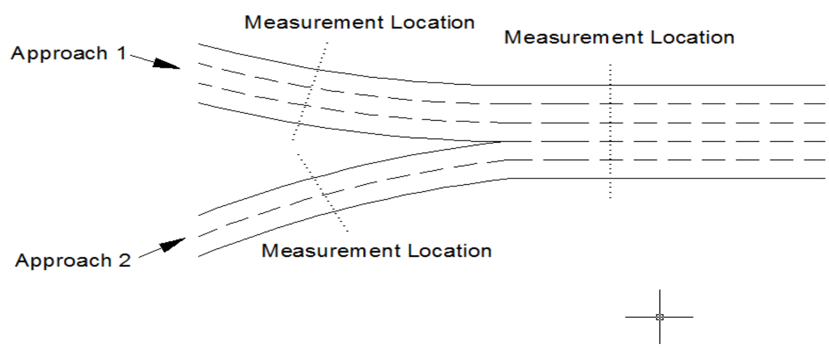
MEASUREMENT METHOD

Historical traffic data from the California Performance Measurement System (PeMS) was used to conduct this study. Study sites were selected based on the following criteria necessary to compute merge ratios: (i) recurrent congestion is present at both upstream approaches and downstream of the merge (i.e. fully congested merges), and (ii) both merging approaches are not metered. For (ii), we only considered freeway-to-freeway merges to ensure that either merging approach was not metered. In addition, two types of merges were used for this study in terms of merge geometry, which we will refer to as type 1 and type 2 merges. In type 1 merges (Fig. 1a), the sum of the number of lanes of both upstream approaches is equal to the number of lanes downstream of the merge. In type 2 merges (Fig. 1b), a lane drop between the upstream and downstream measurement locations exists; i.e. the sum of the lanes of the upstream approaches is greater than the number of lanes downstream of the merge. A total of six merges were selected as study sites; see Table 1.

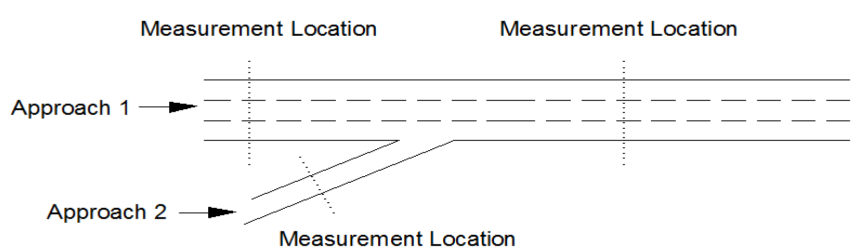
TABLE 1 Summary of study sites

No.	Merge Location	Type	Lanes [a]
1	I10E I405S	1	4/3/1
2	I5S I405S	1	6/3/3
3	SR91W I5N	1	5/3/2
4	US101N SR134W	1	5/3/2
5	I805S SR163S	2	5/4/2
6	I405N SR22W	2	6/4/3

[a] Number of lanes order is downstream/upstream mainline/upstream merging approach



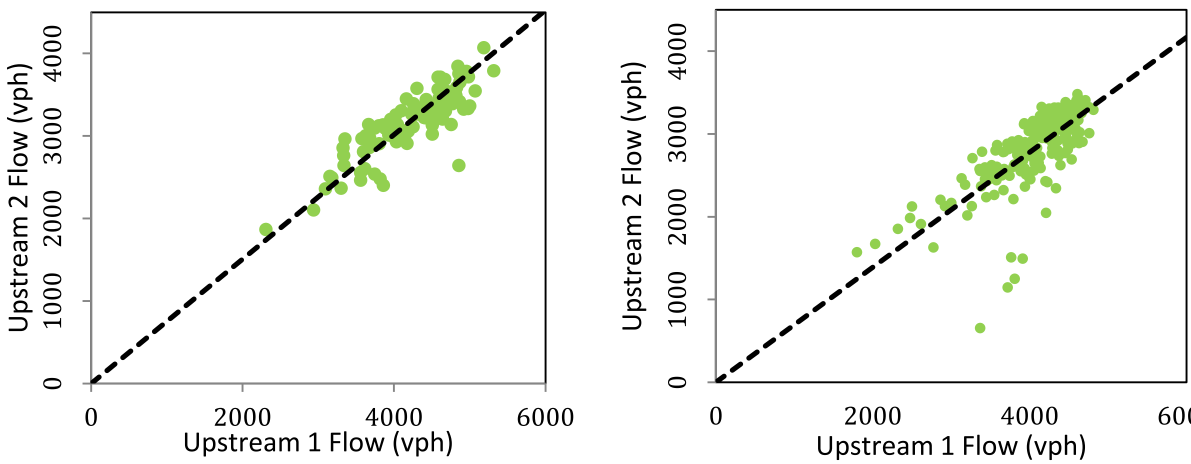
(a)



(b)

Figure 1 Merge schematics, (a) Type 1 merge (b) Type 2 merge

For each study site, we examined two years of 5-minute data to identify fully congested periods and measure average merge ratios. The readers are referred to Bar-Gera and Ahn *(13)* for a detailed description. Based on the preliminary results, congestion events from 6-10 days were selected for a more detailed analysis. On these days, congestion events occurred during the same peak period, assuring recurrent congestion. Moreover, the selected days were free of incidents and unusual weather events. For the selected congested events, 30-second data was used to identify periods of near stationary flow for both upstream and downstream measurement locations. Steady state periods were identified to (i) uncover variations of merge ratio otherwise not captured on 5-minute data and (ii) reduce the scatter in empirical LFD. For this, we adopted the spectral analysis method in Zheng et al. *(23)* and used continuous wavelet transform on oblique cumulative count curves from the upstream mainline approach. The identified periods were assumed to be the same on both upstream approaches. Then flow was measured per steady state period. The average merge ratios computed using the low-resolution and high-resolution data are very similar; however, steady state flows significantly decreased the scatter of the merge ratio plots; see Figure 2 for an example



(a) (b)

Figure 2 Merge ratio of the merge of Interstate 5 north into and State Route 91 west (SR91W I5N), (a) Low-resolution data (b) High-resolution data

During the same steady state periods, we measured LFD, defined as the proportion of flow of each lane with respect to the total flow. Fig. 3 reveals linear trends of LFD against the total flow, as consistent with some previous studies *(17, 20-22)*. Although the linear trends appear moderate, they are statistically significant. This finding suggests that merge ratios may change depending on the merge outflow, although the change may be small enough to disregard for a macroscopic analysis. Observations of LFD at different study sites suggest that LFD is site-specific. For instance, site 3 and site 4 exhibit the same lane configuration. In this case, the estimated merge ratio using fair-share lane ratio would be the same (0.66) for both sites. However, their measured merge ratios are significantly different: 0.75 and 0.61 respectively. It turns out that the two sites exhibit different LFD trends. In site 3, as the total flow increases, the LFD increases in the median lane, decreases in the shoulder lane, and remains fairly constant in the center lanes; Fig. 4a. In contrast, in site 4, the LFD decreases in the median lane, whereas it increases in the lane adjacent to the median lane; see Fig 4b. This observation once again underscores the need for a better estimation model for merge ratio that takes into account site-specific features.

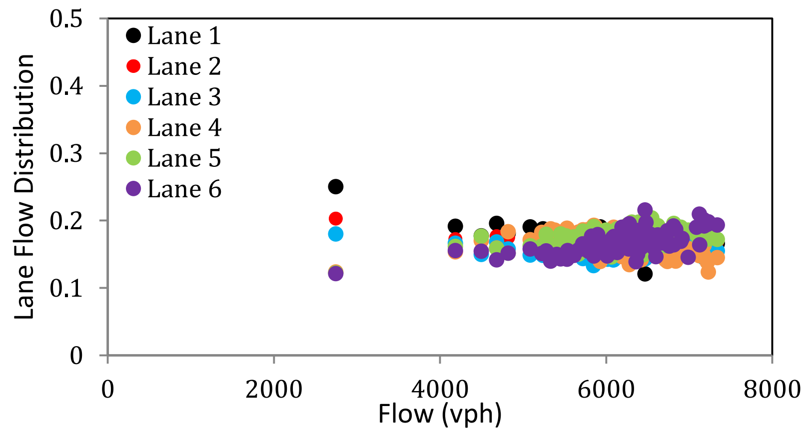
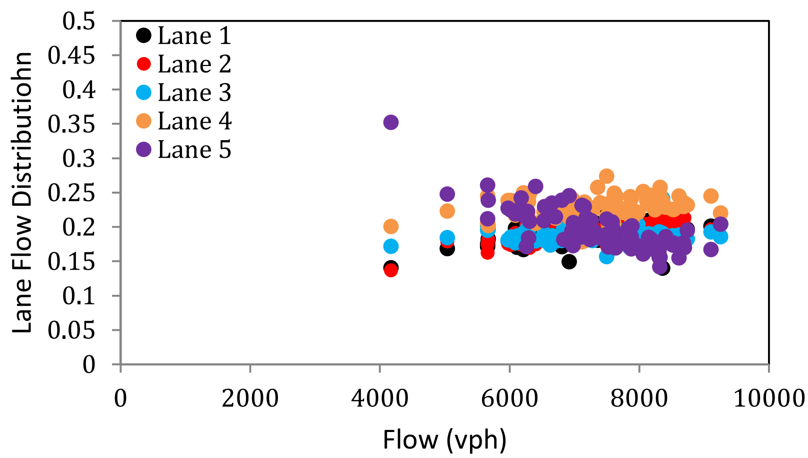
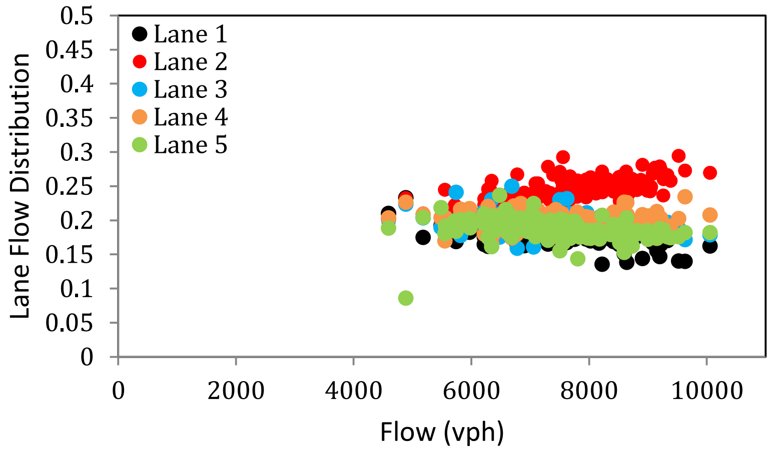


Figure 3 Downstream lane flow distribution versus total flow of the merge of Interstate 405 south into Interstate 5 south (I5S I405S), where lane 1 represents the median lane and the sequence continues towards the shoulder lane



(a)



(b)

Figure 4 Lane flow distribution versus total flow where lane 1 represents the median lane and the sequence continues towards the shoulder lane, (a) SR91W I5N (b) US101N SR134W

FORMULATION OF MERGE RATIOS USING LANE FLOW DISTRIBUTION

In the present study, we define merge ratio in terms of the sum of the LFD in each merging approach. This definition was developed to compute merge ratio using LFD downstream of a merge under both the fair-share and zipper assumptions. Let P_n and P_m represent the LFD of lane n and lane m respectively for a given merge outflow at a location immediately downstream of the merge. Then, the merge ratio under the fair-share assumption is denoted by:

$$\alpha_{fair} = \sum_{n=1}^{N} P_n \Big/ \sum_{m=1}^{M} P_m \tag{1}$$

where N is the total number of lanes on the merging approach (approach 2) and M is the total number of lanes on the mainline freeway approach (approach 1). Similarly, the merge ratio defined in terms of LFD under the zipper rule assumption can be expressed as:

$$\alpha_{zipper} = \left(\sum_{n=1}^{N-1} P_n + \frac{1}{2}\, P_{adj}\right) \Big/ \left(\sum_{m=1}^{M-1} P_m + \frac{1}{2} P_s\right) \tag{2}$$

where P_{adj} is the estimate of the LFD of the lane adjacent to the freeway on the merging approach and P_s is the estimate of the LFD of the shoulder lane on the mainline freeway approach. Note that these formulations are designed to be computed using the LFD measured (or estimated) downstream of the merge, where LFD is not affected by merging behavior. Note that for type 1 merges, it is intuitive that the zipper rule would not apply since vehicles do not have to take turns to merge. Therefore, although we compute merge ratios under both principles for the sake of completeness, we expect the fair-share assumption to hold better for type 1 merges. For type 2 merges, on the other hand, it is not as apparent. Also note that for type 2 merges, one or more lanes in the center should be counted twice.

Preliminary results demonstrate that the proposed formulation represent merge ratios with good accuracy. Figure 5 show the relationship between the merge ratios estimated using LFD and the measured merge ratios. We can observe that they follow a close linear trend with no significant bias. It is also notable that the measured merge ratio does vary to some extent, which would not have been captured with the lane ratio.

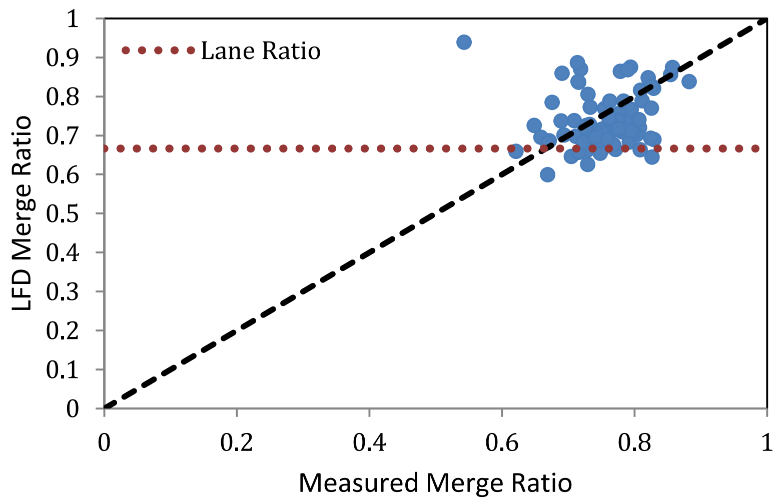


Figure 5 LFD merge ratio versus measured merge ratio under the fair-share principle of site SR91W I5N

TWO-STAGE MODELING OF MERGE RATIO

It is important to realize that computations of merge ratio and LFD are both derived from measurements of flow. Therefore, an inherent correlation between merge ratio and LFD measurements exists. In order for the proposed formulation to have true predictive value, it is necessary to extend the model to efficiently predict LFD based on other variables. To this end, we create a two-stage model structure, where LFD serve as independent variables at the low level, and then become dependent variables at the high level. Specifically, at the low level, LFD are predicted in terms of various characteristics of a freeway. At the high level, merge ratio is predicted in terms of predicted LFD.

To assess the viability of the proposed model structure, preliminary simple linear regression models of LFD for each lane were constructed using flow as the independent variable. Then, the predicted LFD for each lane was used to compute merge ratio using the formulations presented above for each principle. Root-mean-squared-errors (RMSE) were calculated to measure the performance of the models. The results, summarized in Table 2, were compared to those obtained using lane ratios under each premise.

TABLE 2 Summary of root-mean-squared-errors of merge ratio predictions for the preliminary model

No.	Site	Type	Principle	LFD Model	Lane Ratio
1	I10E I405S	1	Fair-Share	**0.037**	0.044
			Zipper	0.144	0.155
2	I5S I405S	1	Fair-Share	**0.086**	0.096
			Zipper	0.090	0.096
3	SR91W I5N	1	Fair-Share	**0.057**	0.105
			Zipper	0.126	0.165
4	US101N SR134W	1	Fair-Share	**0.043**	0.062
			Zipper	0.094	0.050
5	I805S SR163S	2	Fair-Share	0.059	0.100
			Zipper	0.070	**0.053**
6	I405N SR22W	2	Fair-Share	0.087	0.087
			Zipper	0.069	**0.065**

We can observe that for type 1 merges, the lowest RMSE values are obtained for the model based on LFD under the fair-share assumption, as expected. However, the results are somewhat mixed for type 2 merges, where the zipper rule based on the lane ratio applies better, though the LFD-based model performs reasonably well. Overall, the merge ratio model using LFD under the fair-share assumption consistently provides good preliminary estimates of merge ratio. Nevertheless, the preliminary results suggest that there are other factors influencing LFD and thus merge ratio.

As previously discussed, site 3 and site 4 exhibit the same lane geometry, yet they present different merge ratios and different LFD; see Fig. 4 again. Interestingly, site 3 has an off-ramp shortly downstream of the merge, whereas site 4 has an on-ramp. Based on this observation, two new binary variables were incorporated into the linear LFD models: one variable that represents the presence of a nearby downstream on-ramp and another variable that represents the presence of a nearby downstream off-ramp. Furthermore, we develop separate models for facilities with different number of lanes since their LFD would be fundamentally different.

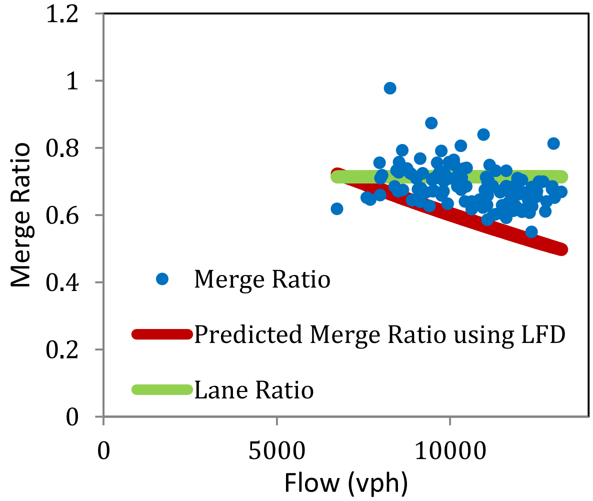
To calibrate the revised models, data from additional sites was included (also from PeMS). These additional sites represent freeways with four, five, and six lanes; and four sites for each freeway type that presented different congested LFD patterns and diverse on-ramp and off-ramp configurations. For each site, a sample of five days was selected following the same rules specified to select days for the merge sites. Note that the presence of downstream on-ramps and off-ramps is defined as an on-ramp or off-ramp; whichever is the closest, located within 0.6 miles downstream of the measurement location. The likelihood ratio test was used for model selection. A total of 20 congested events (leading to 250-350 steady state samples) for each set of models were used to calibrate the models. Table 3 shows the multiple regression model summaries for 4-lane, 5-lane, and 6-lane freeways, including the estimated coefficients, standard errors, and *p*-values. We can observe that most of the variables included in the models are statistical significant at the 0.05 significance level.

TABLE 3 Multiple linear regression parameter estimates

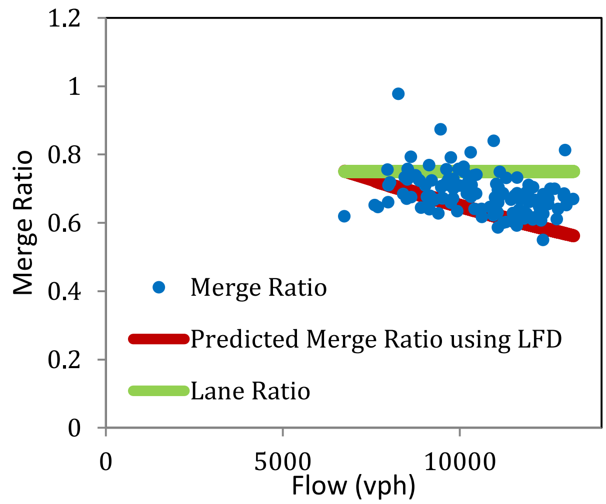
Lane [a]	**Coefficients**	**4-lane Freeway**			**5-lane Freeway**			**6-lane Freeway**		
		Estimate	**Standard Error**	**P-value**	**Estimate**	**Standard Error**	**P-value**	**Estimate**	**Standard Error**	**P-value**
1	(Intercept)	1.59E-01	1.78E-02	<2E-16	8.90E-02	4.06E-03	<2E-16	2.04E-01	1.04E-02	<2E-16
	Flow	1.96E-05	2.92E-06	8.77E-11	1.36E-02	6.68E-07	<2E-16	-2.02E-06	1.20E-06	9.37E-02
	On-ramp	2.09E-02	6.68E-03	1.89E-03	3.49E-02	2.49E-03	<2E-16	-1.36E-02	3.82E-03	4.35E-04
	Off-ramp	-4.40E-02	7.50E-03	1.09E-08	-1.47E-02	3.97E-03	0.000249	2.05E-02	4.67E-03	1.64E-05
2	(Intercept)	2.30E-01	8.08E-03	<2E-16	2.40E-01	2.95E-03	<2E-16	1.65E-01	9.17E-03	<2E-16
	Flow	5.31E-06	1.34E-03	8.80E-05	-4.16E-06	4.86E-07	1.11E-15	1.50E-06	1.05E-06	1.57E-01
	On-ramp	-2.09E-02	3.06E-03	3.86E-11	2.09E-03	1.81E-03	0.2501	-1.52E-02	3.36E-03	9.42E-06
	Off-ramp	-4.41E-02	3.43E-03	<2e-16	8.74E-03	2.88E-03	0.00268	2.51E-02	4.11E-03	3.72E-09
3	(Intercept)	2.67E-01	6.99E-03	<2E-16	2.28E-01	2.42E-03	<2E-16	8.96E-02	7.09E-03	<2E-16
	Flow	-6.21E-06	1.16E-06	1.49E-07	-5.58E-06	3.98E-07	<2E-16	8.90E-06	8.15E-07	<2E-16
	On-ramp	-1.83E-03	2.64E-03	4.89E-01	1.99E-02	1.49E-03	<2E-16	-1.04E-02	2.60E-03	7.72E-05
	Off-ramp	-1.43E-03	2.97E-03	6.31E-01	1.85E-02	2.36E-03	1.44E-13	-1.31E-02	3.17E-03	5.09E-05
4	(Intercept)	3.45E-01	1.60E-02	<2E-16	2.34E-01	2.64E-03	<2E-16	6.21E-02	7.95E-03	1.39E-13
	Flow	-1.87E-05	2.64E-06	9.33E-12	-3.81E-06	4.33E-07	2.37E-16	1.02E-05	9.14E-07	<2E-16
	On-ramp	1.81E-03	6.05E-03	0.765	-1.57E-02	1.62E-03	<2E-16	1.72E-02	2.91E-03	1.04E-08
	Off-ramp	8.96E-02	6.79E-03	<2e-16	-6.50E-03	2.58E-03	0.0123	-5.35E-03	3.56E-03	0.134
5	(Intercept)	N/A			2.09E-01	3.65E-03	<2E-16	1.90E-01	1.66E-02	<2E-16
	Flow				-1.83E-08	6.00E-07	0.9757	-3.86E-06	1.90E-06	0.0435
	On-ramp				-4.12E-02	2.24E-03	<2E-16	5.74E-02	6.06E-03	<2E-16
	Off-ramp				-5.99E-03	3.56E-03	0.0937	-3.91E-03	7.41E-03	0.5982
6	(Intercept)	N/A			N/A			2.90E-01	1.07E-02	<2E-16
	Flow							-1.48E-05	1.24E-06	<2E-16
	On-ramp							-3.54E-02	3.95E-03	<2E-16
	Off-ramp							-2.32E-02	4.83E-03	2.54E-06

[a] Lane 1 represents the median lane and the sequence continues towards the shoulder lane

For validation, we predicted LFD for each lane and for each freeway type using the models above. Then the LFD predictions were used to estimate merge ratios under different rules and compared against the observed values from the original six sites. Fig. 6 shows the results for site 6 as an example. Specifically, Fig. 6a (fair-share) shows the predicted and measured merge ratios vs. the total downstream flow. It is noteworthy that the measured merge ratios exhibit a linear decreasing trend, indicating that the merge ratio varies at this site depending on the traffic condition. Evidently, this variation is not captured by the lane ratio. On the other hand, the fair-share predictions obtained using the LFD estimator follows the linear trend better, although slightly underestimating the merge ratios. We can observe similar trends in Figure 6b (zipper); however, the prediction line does not follow the trend of the measured values as nicely and significantly underestimates the merge ratios.



(a)



(b)

Figure 6 Predictions of merge ratio (I405N SR22W) based on the models, (a) fair-share theory (b) zipper rule

Table 4 compares the overall performance of different models with RMSE as the measure of performance. We can observe that within the LFD-based models, the formulation under the fair-share assumption consistently provides better estimates, with accuracy within 0.1 except for site 2, than the formulation under the zipper principle. Nevertheless, the model based on lane ratio admittedly performs better than the LFD-based models overall. This is attributable to the observation that there are variations in the LFD patterns between sites that have not yet been fully captured by the linear models. However, the fact that the LFD-based models are able to capture variations in merge ratios with respect to the traffic condition and site justifies the effort to enhance the LFD-based models. Efforts to identify better model specifications and other significant factors for LFD are ongoing.

TABLE 4 Summary of root-mean-squared-errors of merge ratio predictions for the preliminary model

No.	Site	Type	Lanes	Principle	LFD Model	Lane Ratio
1	I10E I405S	1	4/3/1	Fair-Share Zipper	0.060 0.163	0.044 0.153
2	I5S I405S	1	6/3/3	Fair-Share Zipper	0.147 0.157	0.096 0.096
3	SR91W I5N	1	5/3/2	Fair-Share Zipper	0.099 0.154	0.105 0.165
4	US101N SR134W	1	5/3/2	Fair-Share Zipper	0.079 0.142	0.062 0.049
5	I805S SR163S	2	5/4/2	Fair-Share Zipper	0.058 0.076	0.105 0.056
6	I405N SR22W	2	6/4/3	Fair-Share Zipper	0.078 0.120	0.087 0.065

CONCLUSIONS

In this study, data from six freeway merges showed that merge ratios can vary between sites even with similar lane configurations. Furthermore, merge ratios can also vary by traffic condition, contrary to previous findings. Therefore, the previous methods to estimate merge ratios based on upstream approaches' capacities or lane ratios are inadequate to capture these variations. Motivated by this finding, we formulated merge ratios based on lane flow distributions (LFD) under the fair-share and zipper principles and examined their capabilities in incorporating the variations and giving better predictions for merge ratios.

We found that the merge ratios estimated based on observed LFD indeed performed better and effectively captured varying merge ratios with respect to site and traffic condition. Taking one step further, we developed two-stage linear models to first estimate LFD with the total freeway flow and presence of on-ramp and off-ramp downstream of a merge as independent variables, and to second estimate merge ratios using the predicted LFD. Data from 12 sites with different geometric configurations were used for calibration, and then the calibrated models were validated against the original data sets. The validation results were promising; the models predicted merge ratios with accuracy within 0.1 in most cases. Yet, the models still need improvement, as they did not perform better than lane ratios in many cases. The authors believe that the LFD models can be improved by enhancing model specification and identifying other significant variables. Further research on this issue is ongoing.

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